

Transportation, Transshipment, and Assignment Problems



Chapter 6

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6-1

Chapter Topics

- The Transportation Model
- Computer Solution of a Transportation Problem
- The Transshipment Model
- The Assignment Model
- Computer Solution of an Assignment Problem

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6-2

Overview

- Part of a class of LP problems known as *network flow models*.
- Special mathematical features that permit very efficient, unique solution methods (variations of traditional simplex procedure).
- Detailed description of methods is contained on the companion website
- Text focuses on model formulation and solution with Excel and QM for windows.
- Web site Module B addresses transportation and assignment solution methods

The Transportation Model: Characteristics

- A product is transported from a number of sources to a number of destinations at the *minimum possible cost*.
- Each *source is able to supply a fixed number* of units of the product, and *each destination has a fixed demand* for the product.
- The linear programming model has *constraints for supply* at each source *and demand* at each destination.
- All constraints are equalities in a balanced transportation model where supply equals demand.
- Constraints contain inequalities in unbalanced models where supply does not equal demand.

Transportation Model Example Problem Definition and Data

How many tons of wheat to transport from each grain elevator to each mill on a monthly basis in order to minimize the total cost of transportation?

<u>Grain Elevator</u>	<u>Supply</u>	<u>Mill</u>	<u>Demand</u>
1. Kansas City	150	A. Chicago	220
2. Omaha	175	B. St. Louis	100
3. Des Moines	275	C. Cincinnati	300
Total	600 tons	Total	600 tons

Transport Cost from Grain Elevator to Mill (\$/ton)

<u>Grain Elevator</u>	<u>A. Chicago</u>	<u>B. St. Louis</u>	<u>C. Cincinnati</u>
1. Kansas City	\$ 6	\$ 8	\$ 10
2. Omaha	7	11	11
3. Des Moines	4	5	12

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6-5

Transportation Model Example Transportation Network Routes

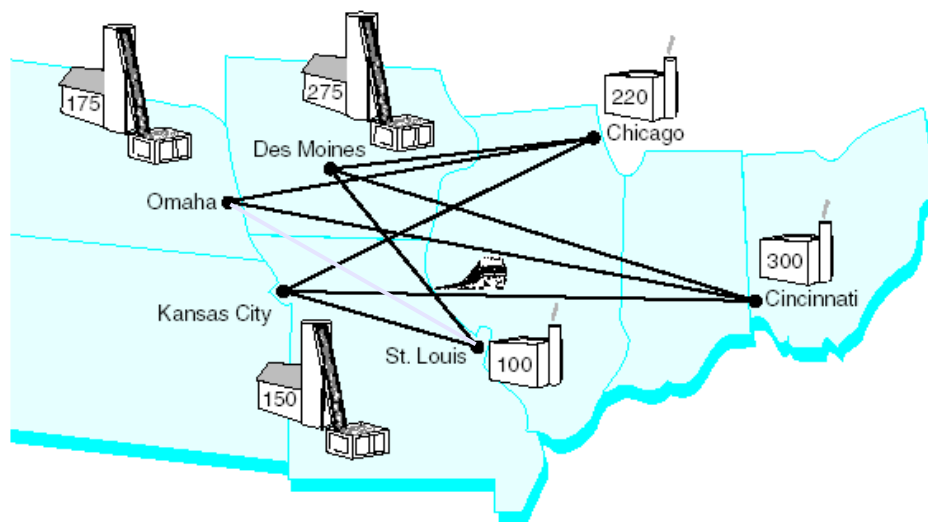


Figure 6.1 Network of transportation routes for wheat shipments

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6-6

Transportation Model Example Model Formulation

$$\text{Minimize } Z = \$6x_{1A} + 8x_{1B} + 10x_{1C} + 7x_{2A} + 11x_{2B} + 11x_{2C} + 4x_{3A} + 5x_{3B} + 12x_{3C}$$

subject to:

$$x_{1A} + x_{1B} + x_{1C} = 150$$

$$x_{2A} + x_{2B} + x_{2C} = 175$$

$$x_{3A} + x_{3B} + x_{3C} = 275$$

$$x_{1A} + x_{2A} + x_{3A} = 200$$

$$x_{1B} + x_{2B} + x_{3B} = 100$$

$$x_{1C} + x_{2C} + x_{3C} = 300$$

$$x_{ij} \geq 0$$

x_{ij} = tons of wheat from each grain elevator, $i, i = 1, 2, 3$,
to each mill $j, j = A, B, C$

Transportation Model Example Computer Solution with Excel (1 of 4)

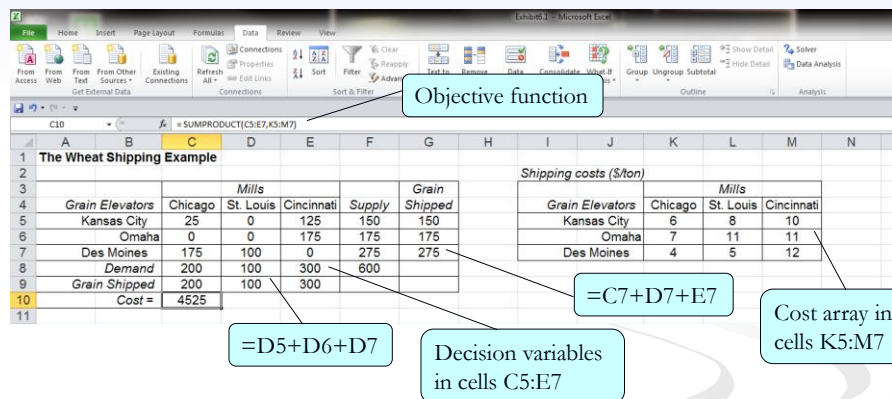


Exhibit 6.1

Transportation Model Example Computer Solution with Excel (2 of 4)

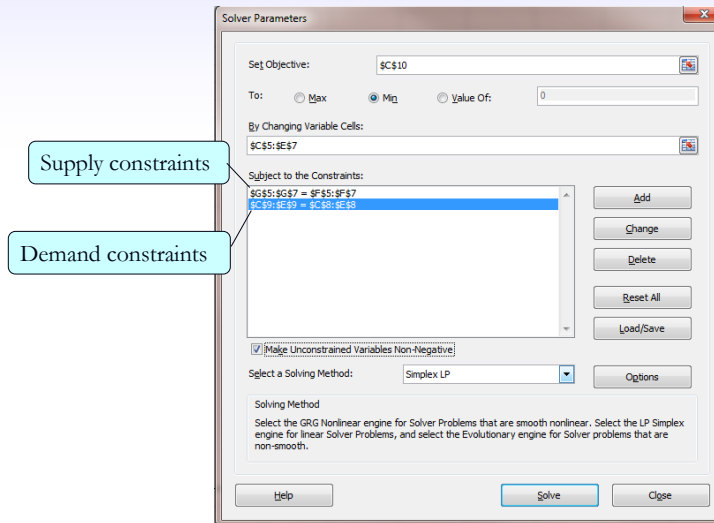


Exhibit 6.2

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6-9

Transportation Model Example Computer Solution with Excel (3 of 4)

Exhibit6.1 [Compatibility Mode] - Microsoft Excel

File

Home

Insert

Page Layout

Formulas

Data

Review

View

From Access

From Web

From Other Source
Get External Data

Existing Connections

Refresh All

Properties

Edit Links

Connections

Sort & Filter

Filter

Clear

Advanced

Text to Columns

Remove Duplicates

Data Validation

Consolidate

What-If Analysis

Group

Ungroup

Subtotal

Outline

</

Exhibit 6.3

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6-10

Transportation Model Example Computer Solution with Excel (4 of 4)

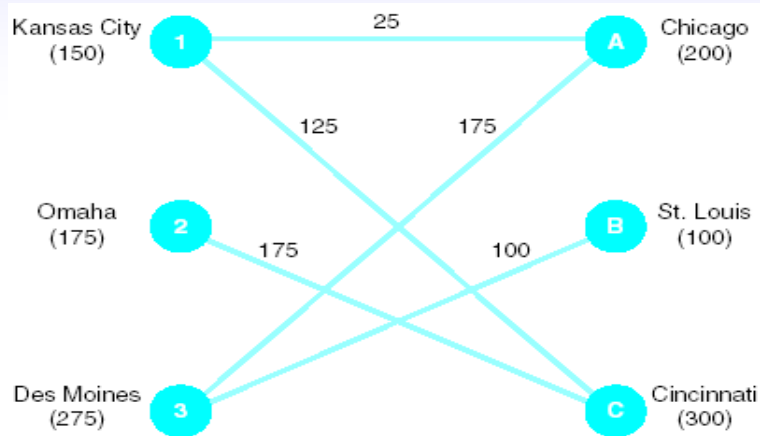


Figure 6.2 Transportation network solution for wheat-shipping example

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6-11

Transportation Model Example Computer Solution with Excel QM (1 of 3)

The dialog box titled "Spreadsheet Initialization" contains the following fields and options:

- Title: Wheat Shipping Example
- Sheet name: (empty)
- Enter the number of origins: 3
- Name for origin: Grain Elevator
- (Use A for A, B, C ... or a for a, b, c ...)
- Enter the number of destinations: 3
- Name for destination: Mill
- Objective:
 - ☐ Maximize
 - ☒ Minimize
- Buttons: Use Default Settings, Help, Cancel, OK

Exhibit 6.4

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6-12

Transportation Model Example Computer Solution with Excel QM (2 of 3)

1. Click on "Add Ins," then "Excel QM" or "Taylor" to access the macro menu

3. Click on "Data," "Solver," and then "Solve."

2. Enter data values for problems; initially this array is blank

Costs	Chicago	St. Louis	Cincinnati	Supply
Kansas City	6	8	10	150
Omaha	7	11	11	175
Des Moines	4	5	12	275
Demand	200	100	300	600/600

Shipments	Chicago	St. Louis	Cincinnati	Row Total
Kansas City				0
Omaha				0
Des Moines				0
Column Total	0	0	0	0/0

Total Cost	
	0

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Exhibit 6.5

6-13

Transportation Model Example Computer Solution with Excel QM (3 of 3)

Click on "Data" tab and then "Solver"

Click on "Data" tab and then "Solver"

Costs	Chicago	St. Louis	Cincinnati	Supply
Kansas City	6	8	10	150
Omaha	7	11	11	175
Des Moines	4	5	12	275
Demand	200	100	300	600/600

Shipments	Chicago	St. Louis	Cincinnati	Row Total
Kansas City	25		150	175
Omaha	175	100		275
Column Total	200	100	300	600/600

Total Cost	
	4525

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Exhibit 6.6

6-14

Transportation Model Example Computer Solution with QM for Windows (1 of 4)

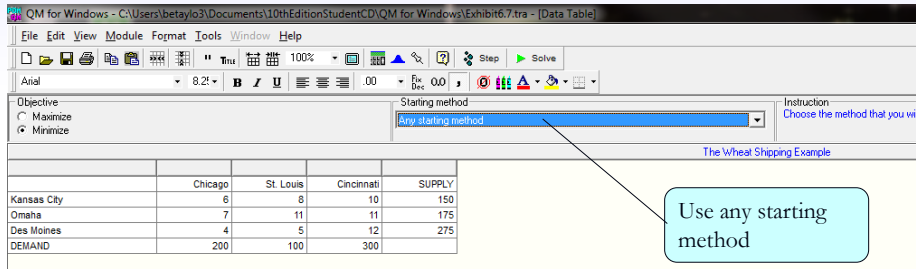


Exhibit 6.7

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6-15

Transportation Model Example Computer Solution with QM for Windows (2 of 4)

Transportation Shipments

The Wheat Shipping Example Solution

	Chicago	St. Louis	Cincinnati
Optimal solution value = \$4,525			
Kansas City			150
Omaha	25		150
Des Moines	175	100	

Exhibit 6.8

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6-16

Transportation Model Example Computer Solution with QM for Windows (3 of 4)

Shipping list

The Wheat Shipping Example Solution

From	To	Shipment	Cost per unit	Shipment cost
Kansas City	Cincinnati	150	10	1,500
Omaha	Chicago	25	7	175
Omaha	Cincinnati	150	11	1,650
Des Moines	Chicago	175	4	700
Des Moines	St. Louis	100	5	500

Exhibit 6.9

Transportation Model Example Computer Solution with QM for Windows (4 of 4)

Exhibit 6.10 - Microsoft Excel

File	Home	Insert	Page Layout	Formulas	Data	Review	View
From Access	From Web	From Text	From Other Sources	Existing Connections	Refresh All	Connections	Properties
			Get External Data				Hyperlinks
							Connections
							Sort
							Filter
							Advanced
							Text to Columns
							Remove Duplicates
							Data Validation
							Consolidate
							What-If Analysis
							Group
							Ungroup
							Subtotal
							Show Detail
							Hide Detail
							Outline
							Analysis

C11

<

Added new row to reflect demand $\geq$ supply

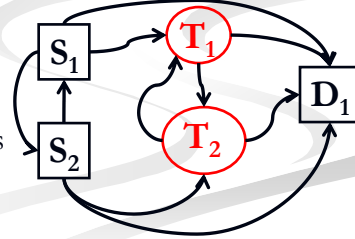
Sensitivity analysis of transportation scenario

Change in cost

Exhibit 6.10

The Transshipment Model Characteristics

- Extension of the transportation model.
- Intermediate transshipment points are added between the sources and destinations.
- Items may be transported from:
 - Sources through transshipment points to destinations
 - One source to another
 - One transshipment point to another
 - One destination to another
 - Directly from sources to destinations
 - Some combination of these



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6-19

Transshipment Model Example Problem Definition and Data

Extension of the transportation model in which intermediate transshipment points are added between sources and destinations.

Shipping Costs

Farm	Grain Elevator		
	3. Kansas City	4. Omaha	5. Des Moines
1. Nebraska	\$16	10	12
2. Colorado	15	14	17

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6-20

Transshipment Model Example

Transshipment Network Routes

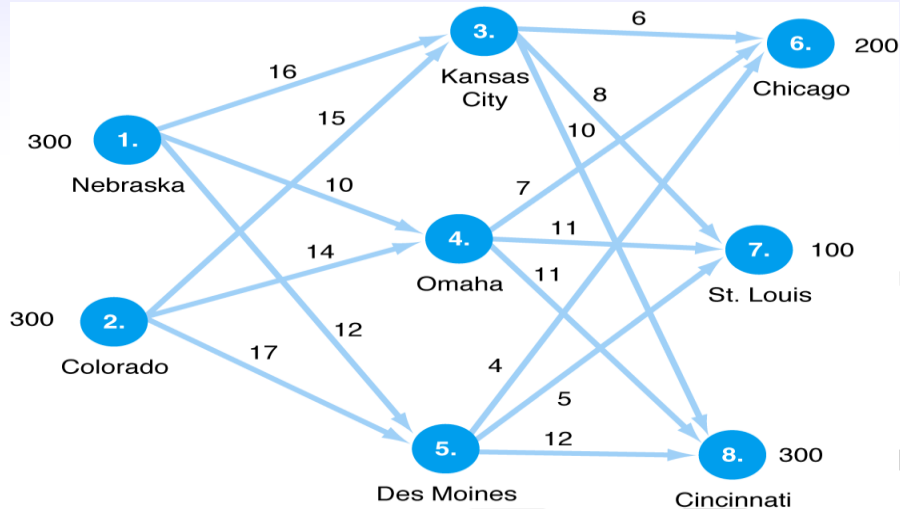


Figure 6.3 Network of transshipment routes

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6-21

Transshipment Model Example

Model Formulation

$$\begin{aligned} \text{Minimize } Z = & \$16x_{13} + 10x_{14} + 12x_{15} + 15x_{23} + 14x_{24} \\ & + 17x_{25} + 6x_{36} + 8x_{37} + 10x_{38} + 7x_{46} + 11x_{47} \\ & + 11x_{48} + 4x_{56} + 5x_{57} + 12x_{58} \end{aligned}$$

subject to:

$$x_{13} + x_{14} + x_{15} = 300$$

$$x_{23} + x_{24} + x_{25} = 300$$

$$x_{36} + x_{46} + x_{56} = 200$$

$$x_{37} + x_{47} + x_{57} = 100$$

$$x_{38} + x_{48} + x_{58} = 300$$

$$x_{13} + x_{23} - x_{36} - x_{37} - x_{38} = 0$$

$$x_{14} + x_{24} - x_{46} - x_{47} - x_{48} = 0$$

$$x_{15} + x_{25} - x_{56} - x_{57} - x_{58} = 0$$

$$x_{ij} \geq 0$$

Supply constraints for farms
in Nebraska and Colorado

Demand constraints at
the Chicago, St. Louis
and Cincinnati mills

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Transshipment Model Example Computer Solution with Excel (1 of 3)

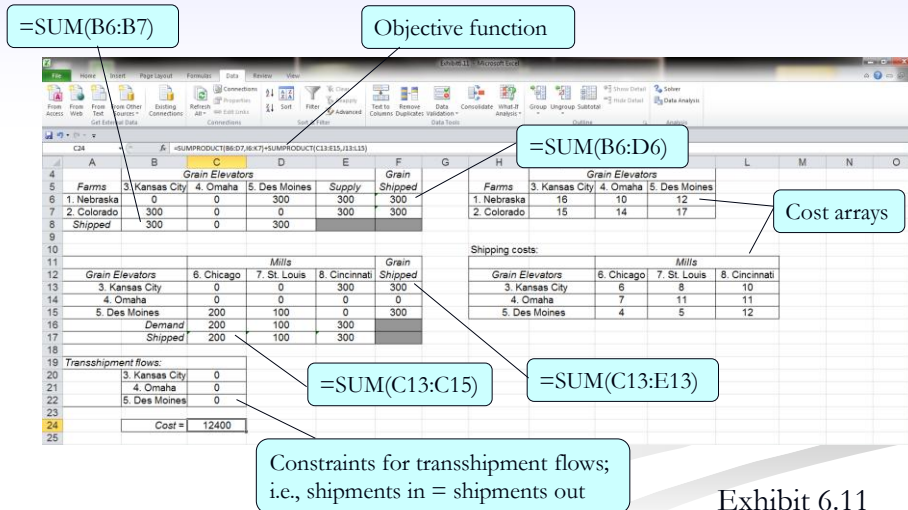


Exhibit 6.11

Transshipment Model Example Computer Solution with Excel (2 of 3)

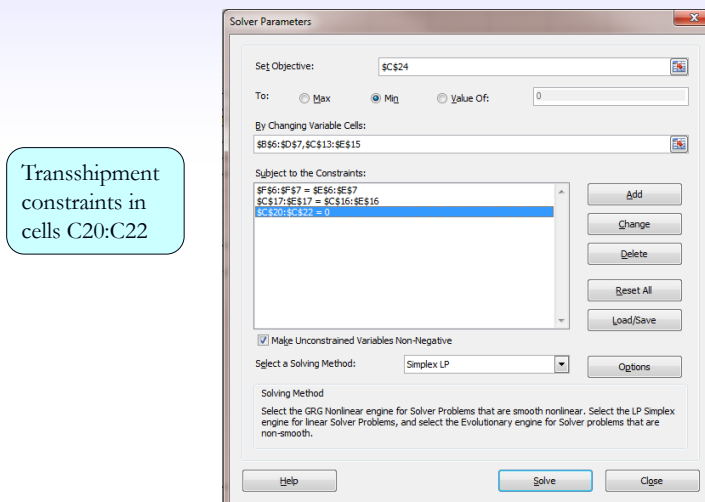


Exhibit 6.12

Transshipment Model Example

Network Solution for Wheat Shipping (3 of 3)

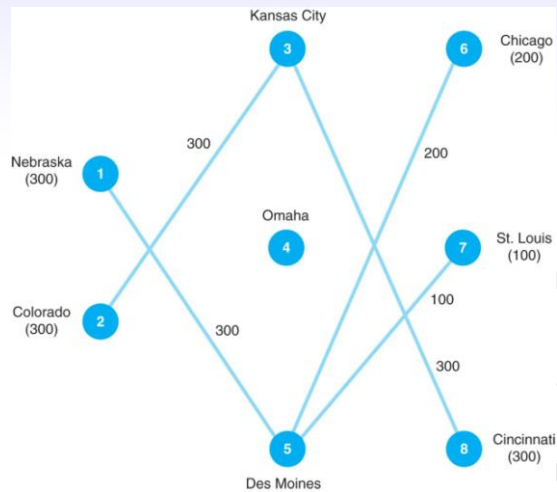


Figure 6.4 Transshipment network solution for wheat-shipping example

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6-25

The Assignment Model

Characteristics

- Special form of linear programming model similar to the transportation model.
- **Supply** at each source **and demand** at each destination **limited to one unit.**
- In a balanced model supply equals demand.
- In an unbalanced model supply does not equal demand.

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6-26

Assignment Model Example Problem Definition and Data

Problem: Assign four teams of officials to four games in a way that will minimize total distance traveled by the officials. Supply is always one team of officials, demand is for only one team of officials at each game.

Officials	Game Sites			
	RALEIGH	ATLANTA	DURHAM	CLEMSON
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

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6-27

Assignment Model Example Model Formulation

$$\begin{aligned} \text{Minimize } Z = & 210x_{AR} + 90x_{AA} + 180x_{AD} + 160x_{AC} + 100x_{BR} + 70x_{BA} \\ & + 130x_{BD} + 200x_{BC} + 175x_{CR} + 105x_{CA} + 140x_{CD} \\ & + 170x_{CC} + 80x_{DR} + 65x_{DA} + 105x_{DD} + 120x_{DC} \end{aligned}$$

subject to:

$$\begin{aligned} x_{AR} + x_{AA} + x_{AD} + x_{AC} &= 1 & x_{ij} &\geq 0 \\ x_{BR} + x_{BA} + x_{BD} + x_{BC} &= 1 \\ x_{CR} + x_{CA} + x_{CD} + x_{CC} &= 1 \\ x_{DR} + x_{DA} + x_{DD} + x_{DC} &= 1 \\ x_{AR} + x_{BR} + x_{CR} + x_{DR} &= 1 \\ x_{AA} + x_{BA} + x_{CA} + x_{DA} &= 1 \\ x_{AD} + x_{BD} + x_{CD} + x_{DD} &= 1 \\ x_{AC} + x_{BC} + x_{CC} + x_{DC} &= 1 \end{aligned}$$

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6-28

Assignment Model Example Computer Solution with Excel (1 of 3)

Objective function

Decision variables, C5:F8

=C5+D5+E5+F5

=D5+D6+D7+D8

Mileage array

The ACC Basketball Example						
	Game Sites				Teams Available	Teams Assigned
Officials Team	Raleigh	Atlanta	Durham	Clemson		
A					1	0
B					1	0
C					1	0
D					1	0
Teams Demanded	1	1	1	1	4	4
Teams Assigned	0	0	0	0		
Total Mileage =						

Mileage:				
Officials Team	Raleigh	Atlanta	Durham	Clemson
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

Exhibit 6.13

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6-29

Assignment Model Example Computer Solution with Excel (2 of 3)

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

\$H\$5:\$H\$8 = \$G\$5:\$G\$8
\$C\$10:\$F\$10 = \$C\$9:\$F\$9

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method:

Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Buttons: Add, Change, Delete, Reset All, Load/Save, Options, Help, Solve, Close

Exhibit 6.14

Simplex LP

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6-30

Assignment Model Example Computer Solution with Excel QM

Enter the assignment costs in the shaded area. Then go to the DATA Tab on the ribbon, click on Solver in the Data Analysis Group and then click SOLVE. If SOLVER is not on the Data Tab then please see the Help file

Mileage	Raleigh	Atlanta	Durham	Clemson
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	85	105	120

Officials/To	Raleigh	Atlanta	Durham	Clemson	Row Total
A		1			1
B	1				1
C			1		1
D				1	1
Column Total	1	1	1	1	4

	Raleigh	Atlanta	Durham	Clemson
Total Cost	450			

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Exhibit 6.16

6-33

Assignment Model Example Computer Solution with QM for Windows (1 of 2)

Objective: ☐ Maximize ☒ Minimize

Instruction: Enter the cost of assigning d to clemson. Any real value is permissible.

	Raleigh	Atlanta	Durham	Clemson
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	85	105	120

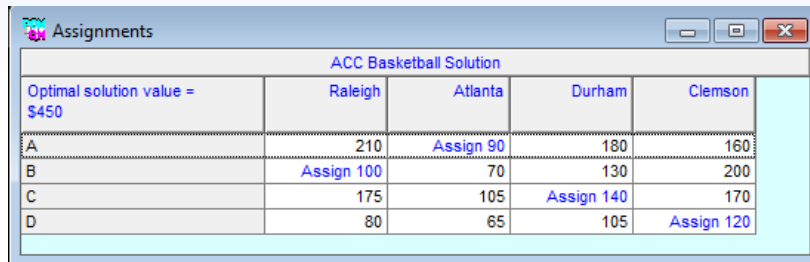
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Exhibit 6.17

6-34

Assignment Model Example

Computer Solution with QM for Windows (2 of 2)



	Raleigh	Atlanta	Durham	Clemson
Optimal solution value = \$450				
A	210	Assign 90	180	160
B	Assign 100	70	130	200
C	175	105	Assign 140	170
D	80	65	105	Assign 120

Exhibit 6.18

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6-35

Example Problem Solution

Transportation Problem Statement

A concrete company transports concrete from three plants to three construction sites. The supply capacities of the three plants, the demand requirements at the three sites, and the transportation costs per ton are as follows:

Plant	Construction site			Supply (tons)
	A	B	C	
1	\$ 8	\$ 5	\$ 6	120
2	15	10	12	80
3	3	9	10	80
Demand (tons)	150	70	100	

Determine the linear programming model formulation and solve using Excel.

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6-36

Example Problem Solution Model Formulation

$$\text{Minimize } Z = \$8x_{1A} + 5x_{1B} + 6x_{1C} + 15x_{2A} + 10x_{2B} + 12x_{2C} \\ + 3x_{3A} + 9x_{3B} + 10x_{3C}$$

subject to:

$$x_{1A} + x_{1B} + x_{1C} = 120$$

$$x_{2A} + x_{2B} + x_{2C} = 80$$

$$x_{3A} + x_{3B} + x_{3C} = 80$$

$$x_{1A} + x_{2A} + x_{3A} \leq 150$$

$$x_{1B} + x_{2B} + x_{3B} \leq 70$$

$$x_{1C} + x_{2C} + x_{3C} \leq 100$$

$$x_{ij} \geq 0$$

Example Problem Solution Computer Solution with Excel

The screenshot shows an Excel spreadsheet titled "Chapter 6-Example - Microsoft". The formula bar shows the formula for cell B10: $=8*B5+5*C5+6*D5+15*B6+10*C6+12*D6+3*B7+9*C7+10*D7$.

	A	B	C	D	E	F	G	H	I	J
1	Example Problem									
2										
3		Construction Site								
4	Plant	A	B	C	Supply	Transported				
5	1	70	30	20	120	120				
6	2	80	0	0	80	80				
7	3	0	0	80	80	80				
8	Demand	150	70	100						
9	Transported	150	30	100						
10	Cost =	2830								
11										



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