Transportation, Transshipment, and Assignment Problems

Chapter 6

Chapter Topics

- The Transportation Model
- Computer Solution of a Transportation Problem
- The Transshipment Model
- The Assignment Model
- Computer Solution of an Assignment Problem

Overview

- Part of a class of LP problems known as ***network flow models***.
- Special mathematical features that permit very efficient, unique solution methods (variations of traditional simplex procedure).
- Detailed description of methods is contained on the companion website
- Text focuses on model formulation and solution with Excel and QM for windows.
- Web site Module B addresses transportation and assignment solution methods

The Transportation Model: Characteristics

- A product is transported from a number of sources to a number of destinations at the ***minimum possible cost***.
- Each ***source is able to supply a fixed number*** of units of the product, and ***each destination has a fixed demand*** for the product.
- The linear programming model has ***constraints for supply*** at each source ***and demand*** at each destination.
- All constraints are equalities in a balanced transportation model where supply equals demand.
- Constraints contain inequalities in unbalanced models where supply does not equal demand.

Transportation Model Example
Problem Definition and Data

How many tons of wheat to transport from each grain elevator to each mill on a monthly basis in order to minimize the total cost of transportation?

Grain Elevator	Supply	Mill	Demand
1. Kansas City	150	A. Chicago	220
2. Omaha	175	B. St. Louis	100
3. Des Moines	275	C. Cincinnati	300
Total	600 tons	Total	600 tons

Transport Cost from Grain Elevator to Mill ($/ton)

Grain Elevator	A. Chicago	B. St. Louis	C. Cincinnati
1. Kansas City	$ 6	$ 8	$ 10
2. Omaha	7	11	11
3. Des Moines	4	5	12

Transportation Model Example
Transportation Network Routes

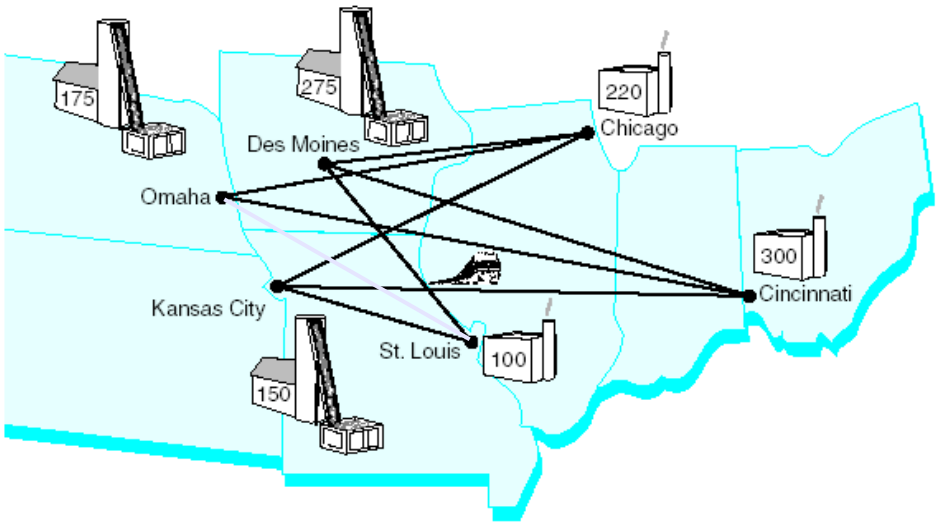


Figure 6.1 Network of transportation routes for wheat shipments

Transportation Model Example
Model Formulation

Minimize $Z = \$6x_{1A} + 8x_{1B} + 10x_{1C} + 7x_{2A} + 11x_{2B} + 11x_{2C} + 4x_{3A} + 5x_{3B} + 12x_{3C}$

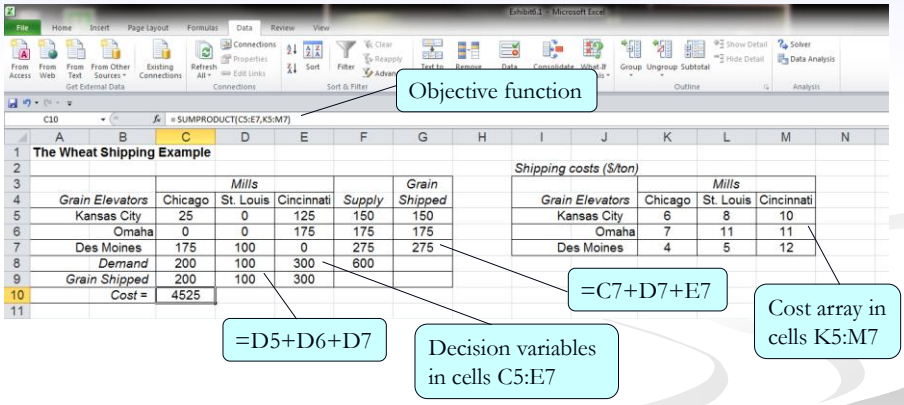
subject to:

$$x_{1A} + x_{1B} + x_{1C} = 150$$
$$x_{2A} + x_{2B} + x_{2C} = 175$$
$$x_{3A} + x_{3B} + x_{3C} = 275$$
$$x_{1A} + x_{2A} + x_{3A} = 200$$
$$x_{1B} + x_{2B} + x_{3B} = 100$$
$$x_{1C} + x_{2C} + x_{3C} = 300$$
$$x_{ij} \geq 0$$

x_{ij} = tons of wheat from each grain elevator, i, i = 1, 2, 3,

to each mill j, j = A,B,C

Transportation Model Example
Computer Solution with Excel (1 of 4)



	A	B	C	D	E	F	G
1	The Wheat Shipping Example						
2							
3			Mills				Grain
4	Grain Elevators		Chicago	St. Louis	Cincinnati	Supply	Shipped
5	Kansas City		25	0	125	150	150
6	Omaha		0	0	175	175	175
7	Des Moines		175	100	0	275	275
8	Demand		200	100	300	600	
9	Grain Shipped		200	100	300		
10	Cost =		4525				
11							

	I	J	K	L	M
	Shipping costs ($/ton)				
			Mills		
	Grain Elevators		Chicago	St. Louis	Cincinnati
	Kansas City		6	8	10
	Omaha		7	11	11
	Des Moines		4	5	12

Exhibit 6.1

Transportation Model Example
Computer Solution with Excel (2 of 4)

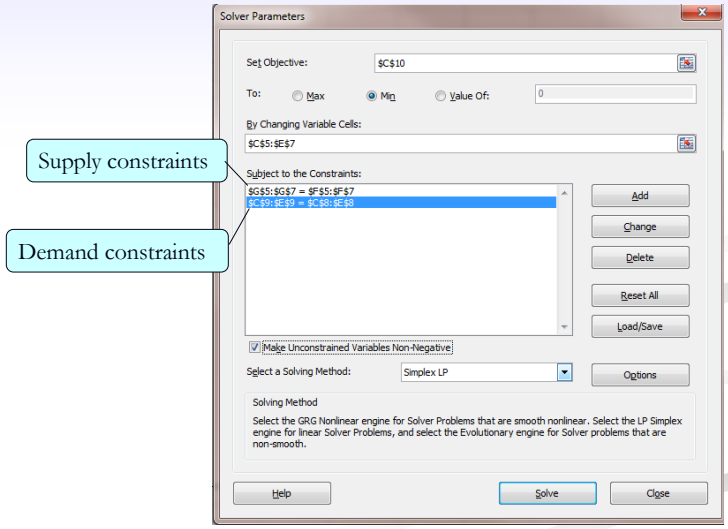
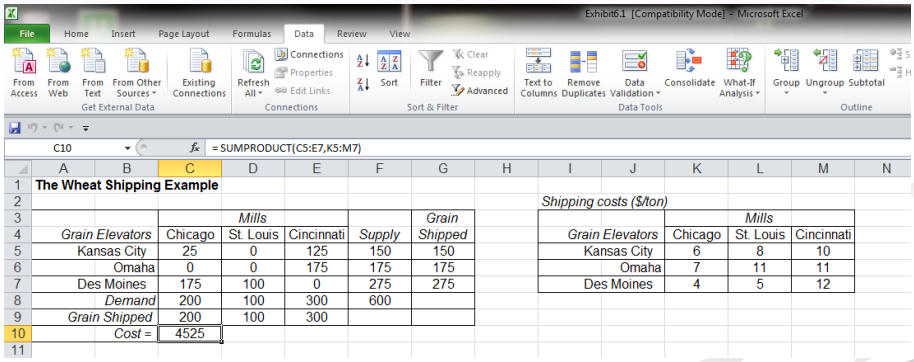


Exhibit 6.2

Transportation Model Example
Computer Solution with Excel (3 of 4)



	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	**The Wheat Shipping Example**													
2									*Shipping costs ($/ton)*					
3				*Mills*			*Grain*					*Mills*		
4		*Grain Elevators*	Chicago	St. Louis	Cincinnati	*Supply*	*Shipped*		*Grain Elevators*		Chicago	St. Louis	Cincinnati	
5		Kansas City	25	0	125	150	150		Kansas City		6	8	10	
6		Omaha	0	0	175	175	175		Omaha		7	11	11	
7		Des Moines	175	100	0	275	275		Des Moines		4	5	12	
8		*Demand*	200	100	300	600								
9		*Grain Shipped*	200	100	300									
10		*Cost =*	4525											
11														

Exhibit 6.3

Transportation Model Example
Computer Solution with Excel (4 of 4)

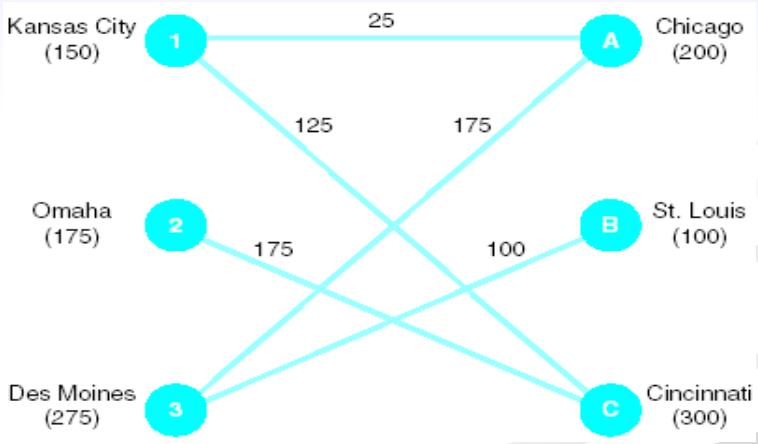


Figure 6.2 Transportation network solution for wheat-shipping example

Transportation Model Example
Computer Solution with Excel QM (1 of 3)

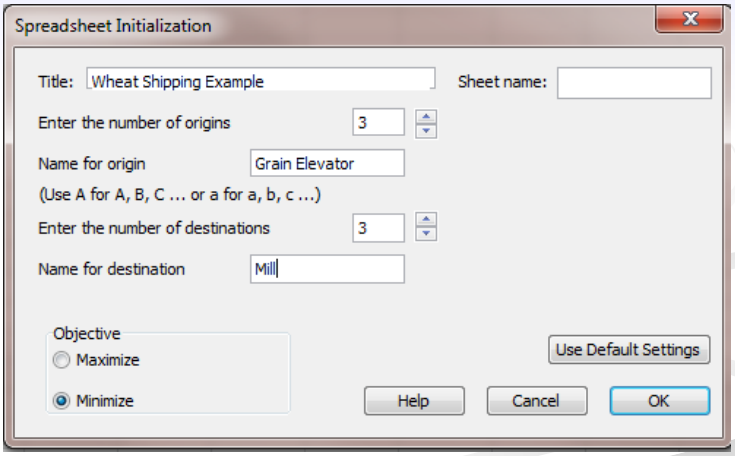
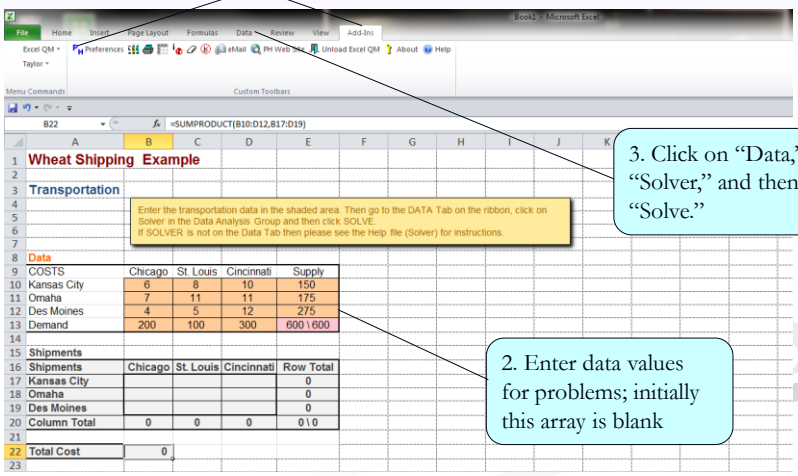


Exhibit 6.4

Transportation Model Example
Computer Solution with Excel QM (2 of 3)

1. Click on "Add Ins," then "Excel QM" or "Taylor" to access the macro menu



3. Click on "Data," "Solver," and then "Solve."

2. Enter data values for problems; initially this array is blank

Exhibit 6.5

Transportation Model Example
Computer Solution with Excel QM (3 of 3)

Click on "Data" tab and then "Solver"

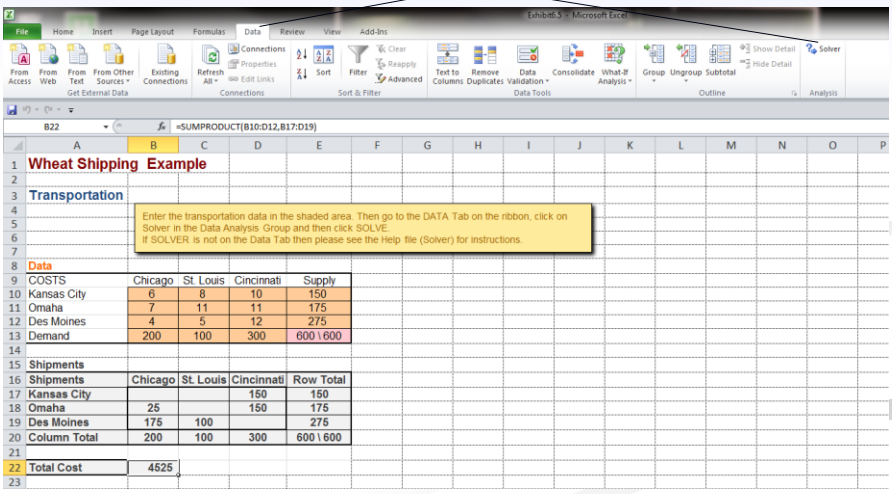
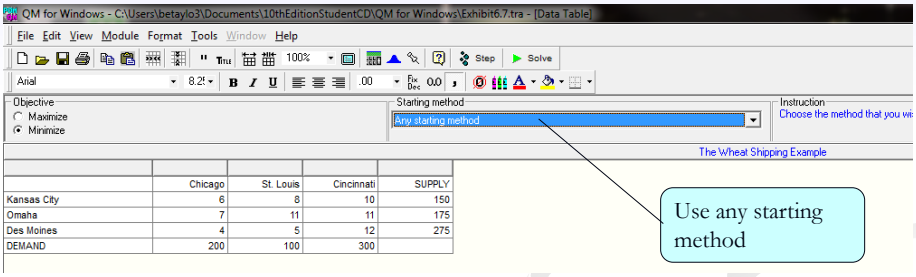


Exhibit 6.6

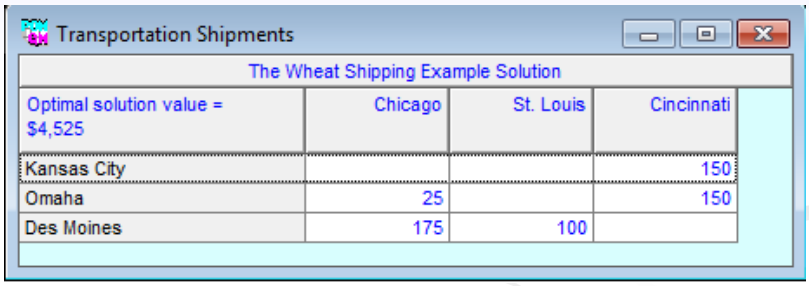
Transportation Model Example
Computer Solution with QM for Windows (1 of 4)



Use any starting method

Exhibit 6.7

Transportation Model Example
Computer Solution with QM for Windows (2 of 4)



Optimal solution value = $4,525	Chicago	St. Louis	Cincinnati
Kansas City			150
Omaha	25		150
Des Moines	175	100	

Exhibit 6.8

Transportation Model Example
Computer Solution with QM for Windows (3 of 4)

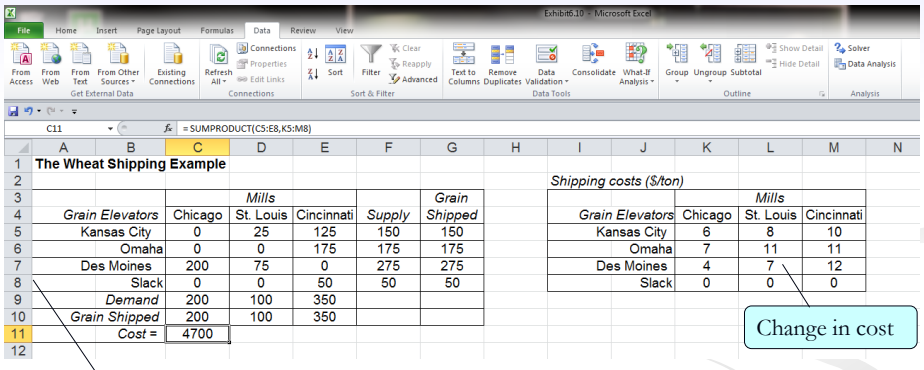
Shipping list

The Wheat Shipping Example Solution

From	To	Shipment	Cost per unit	Shipment cost
Kansas City	Cincinnati	150	10	1,500
Omaha	Chicago	25	7	175
Omaha	Cincinnati	150	11	1,650
Des Moines	Chicago	175	4	700
Des Moines	St. Louis	100	5	500

Exhibit 6.9

Transportation Model Example
Computer Solution with QM for Windows (4 of 4)



C11 = SUMPRODUCT(C5:E8,K5:M8)

	A	B	C	D	E	F	G
1	The Wheat Shipping Example						
2							
3				Mills			Grain
4		Grain Elevators	Chicago	St. Louis	Cincinnati	Supply	Shipped
5		Kansas City	0	25	125	150	150
6		Omaha	0	0	175	175	175
7		Des Moines	200	75	0	275	275
8		Slack	0	0	50	50	50
9		Demand	200	100	350		
10		Grain Shipped	200	100	350		
11		Cost =	4700				
12							

Shipping costs ($/ton)

Grain Elevators	Chicago	St. Louis	Cincinnati
		Mills	
Kansas City	6	8	10
Omaha	7	11	11
Des Moines	4	7	12
Slack	0	0	0

Change in cost

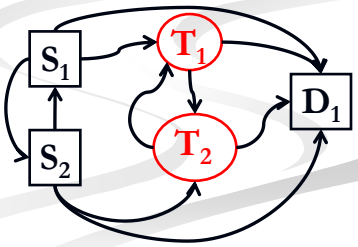
Added new row to reflect demand ≥ supply

Sensitivity analysis of transportation scenario

Exhibit 6.10

The Transshipment Model Characteristics

- Extension of the transportation model.
- Intermediate transshipment points are added between the sources and destinations.
- Items may be transported from:
  - Sources through transshipment points to destinations
  - One source to another
  - One transshipment point to another
  - One destination to another
  - Directly from sources to destinations
  - Some combination of these



Transshipment Model Example Problem Definition and Data

Extension of the transportation model in which intermediate transshipment points are added between sources and destinations.

Shipping Costs

	Grain Elevator		
Farm	3. Kansas City	4. Omaha	5. Des Moines
1. Nebraska	$16	10	12
2. Colorado	15	14	17

Transshipment Model Example
Transshipment Network Routes

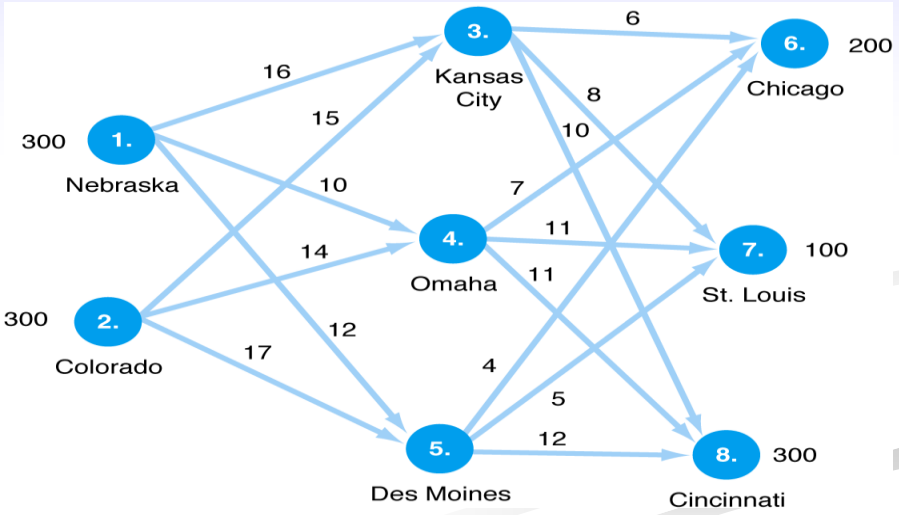


Figure 6.3 Network of transshipment routes

Transshipment Model Example
Model Formulation

Minimize Z = $16x_{13} + 10x_{14} + 12x_{15} + 15x_{23} + 14x_{24}$
$+ 17x_{25} + 6x_{36} + 8x_{37} + 10x_{38} + 7x_{46} + 11x_{47}$
$+ 11x_{48} + 4x_{56} + 5x_{57} + 12x_{58}$

subject to:

$x_{13} + x_{14} + x_{15} = 300$
$x_{23} + x_{24} + x_{25} = 300$

Supply constraints for farms in Nebraska and Colorado

$x_{36} + x_{46} + x_{56} = 200$
$x_{37} + x_{47} + x_{57} = 100$
$x_{38} + x_{48} + x_{58} = 300$

Demand constraints at the Chicago, St. Louis and Cincinnati mills

$x_{13} + x_{23} - x_{36} - x_{37} - x_{38} = 0$
$x_{14} + x_{24} - x_{46} - x_{47} - x_{48} = 0$
$x_{15} + x_{25} - x_{56} - x_{57} - x_{58} = 0$
$x_{ij} \geq 0$

Transshipment Model Example
Computer Solution with Excel (1 of 3)

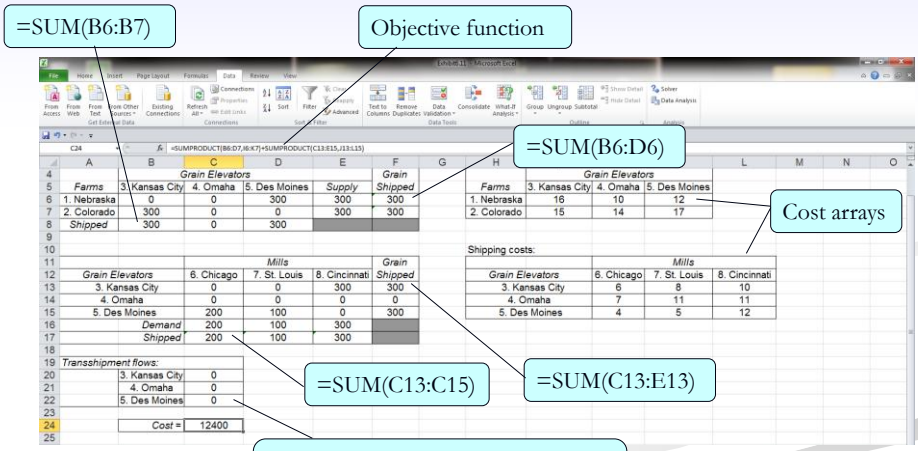


Exhibit 6.11

Transshipment Model Example
Computer Solution with Excel (2 of 3)

Transshipment constraints in cells C20:C22

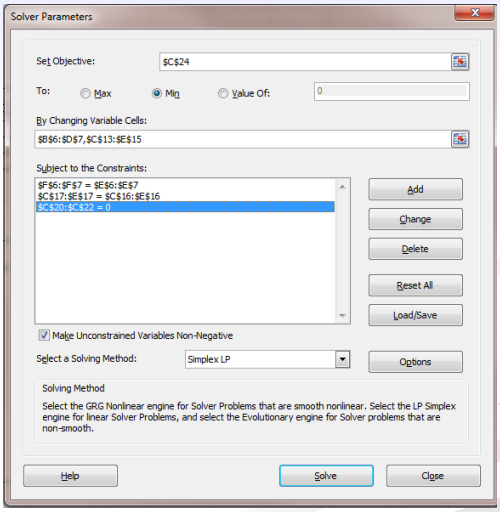


Exhibit 6.12

Transshipment Model Example
Network Solution for Wheat Shipping (3 of 3)

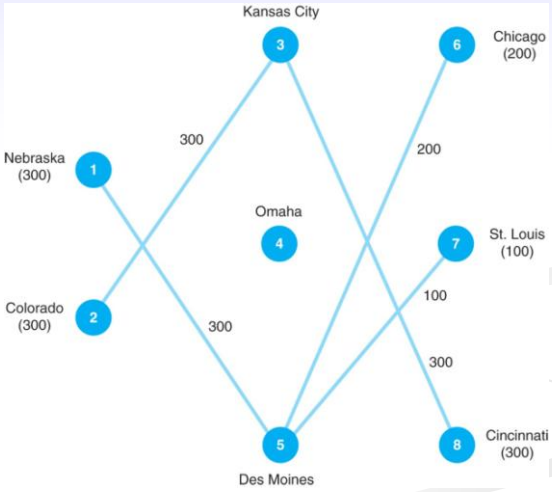


Figure 6.4 Transshipment network solution for wheat-shipping example

The Assignment Model Characteristics

- Special form of linear programming model similar to the transportation model.
- ***Supply*** at each source ***and demand*** at each destination ***limited to one unit***.
- In a balanced model supply equals demand.
- In an unbalanced model supply does not equal demand.

Assignment Model Example
Problem Definition and Data

Problem: Assign four teams of officials to four games in a way that will minimize total distance traveled by the officials. Supply is always one team of officials, demand is for only one team of officials at each game.

	Game Sites			
Officials	RALEIGH	ATLANTA	DURHAM	CLEMSON
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

Assignment Model Example
Model Formulation

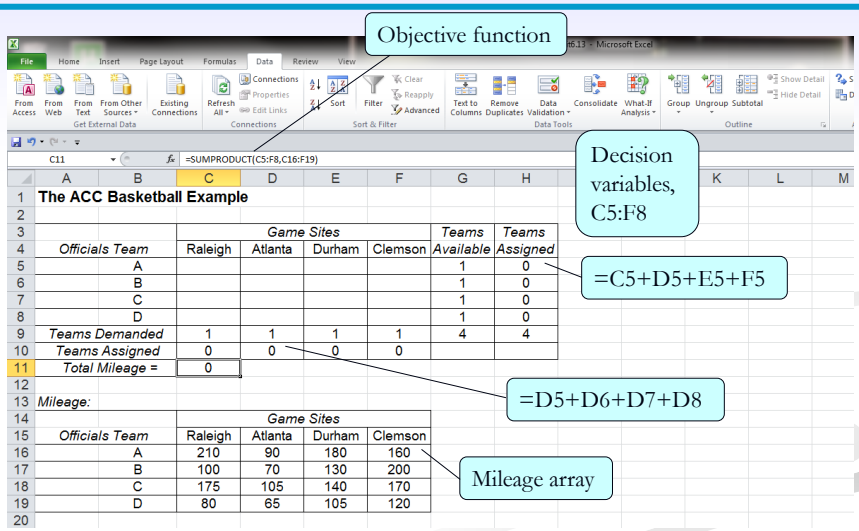
Minimize Z = $210x_{AR} + 90x_{AA} + 180x_{AD} + 160x_{AC} + 100x_{BR} + 70x_{BA} + 130x_{BD} + 200x_{BC} + 175x_{CR} + 105x_{CA} + 140x_{CD} + 170x_{CC} + 80x_{DR} + 65x_{DA} + 105x_{DD} + 120x_{DC}$

subject to:

$x_{AR} + x_{AA} + x_{AD} + x_{AC} = 1$
$x_{BR} + x_{BA} + x_{BD} + x_{BC} = 1$
$x_{CR} + x_{CA} + x_{CD} + x_{CC} = 1$
$x_{DR} + x_{DA} + x_{DD} + x_{DC} = 1$
$x_{AR} + x_{BR} + x_{CR} + x_{DR} = 1$
$x_{AA} + x_{BA} + x_{CA} + x_{DA} = 1$
$x_{AD} + x_{BD} + x_{CD} + x_{DD} = 1$
$x_{AC} + x_{BC} + x_{CC} + x_{DC} = 1$

$x_{ij} \geq 0$

Assignment Model Example
Computer Solution with Excel (1 of 3)



	A	B	C	D	E	F	G	H
1	**The ACC Basketball Example**							
2								
3			*Game Sites*				*Teams*	*Teams*
4		*Officials Team*	Raleigh	Atlanta	Durham	Clemson	*Available*	*Assigned*
5		A					1	0
6		B					1	0
7		C					1	0
8		D					1	0
9		*Teams Demanded*	1	1	1	1	4	4
10		*Teams Assigned*	0	0	0	0		
11		*Total Mileage =*	0					
12								
13	*Mileage:*							
14			*Game Sites*					
15		*Officials Team*	Raleigh	Atlanta	Durham	Clemson		
16		A	210	90	180	160		
17		B	100	70	130	200		
18		C	175	105	140	170		
19		D	80	65	105	120		
20								

Exhibit 6.13

Assignment Model Example
Computer Solution with Excel (2 of 3)

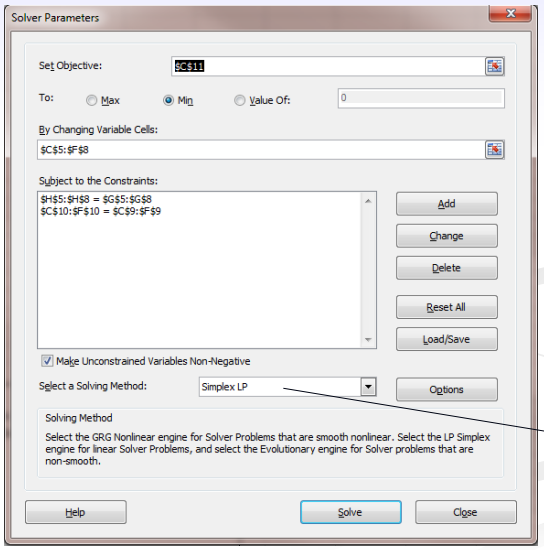
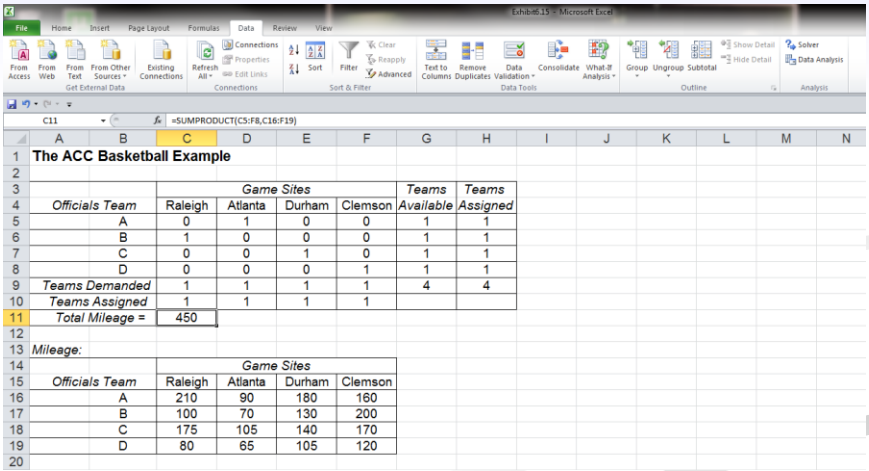


Exhibit 6.14

Assignment Model Example
Computer Solution with Excel (3 of 3)



=SUMPRODUCT(C5:F8,C16:F19)

	A	B	C	D	E	F	G	H
1	**The ACC Basketball Example**							
2								
3			*Game Sites*				*Teams*	*Teams*
4	*Officials Team*		Raleigh	Atlanta	Durham	Clemson	*Available*	*Assigned*
5		A	0	1	0	0	1	1
6		B	1	0	0	0	1	1
7		C	0	0	1	0	1	1
8		D	0	0	0	1	1	1
9	*Teams Demanded*		1	1	1	1	4	4
10	*Teams Assigned*		1	1	1	1		
11	*Total Mileage =*		450					
12								
13	*Mileage:*							
14			*Game Sites*					
15	*Officials Team*		Raleigh	Atlanta	Durham	Clemson		
16		A	210	90	180	160		
17		B	100	70	130	200		
18		C	175	105	140	170		
19		D	80	65	105	120		
20								

Exhibit 6.15

Assignment Model Example
Assignment Network Solution

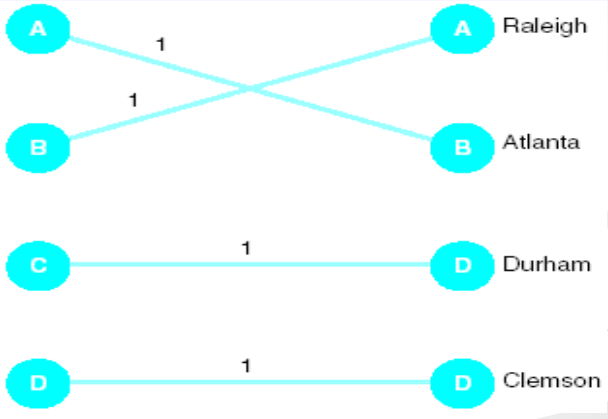


Figure 6.5 Assignment network solution for ACC officials

Assignment Model Example
Computer Solution with Excel QM

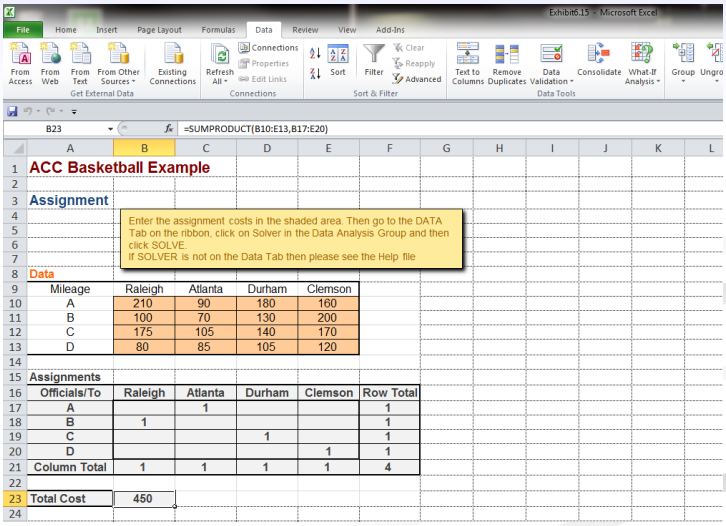


Exhibit 6.16

Assignment Model Example
Computer Solution with QM for Windows (1 of 2)

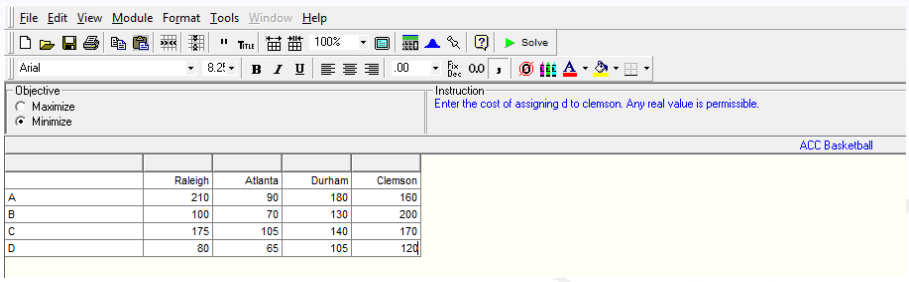
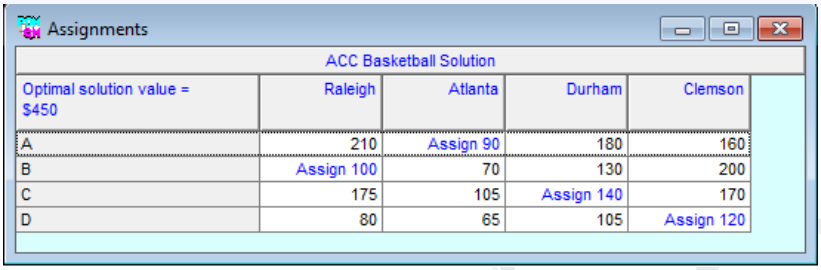


Exhibit 6.17

Assignment Model Example
Computer Solution with QM for Windows (2 of 2)



Optimal solution value = $450	Raleigh	Atlanta	Durham	Clemson
A	210	Assign 90	180	160
B	Assign 100	70	130	200
C	175	105	Assign 140	170
D	80	65	105	Assign 120

Exhibit 6.18

Example Problem Solution
Transportation Problem Statement

A concrete company transports concrete from three plants to three construction sites. The supply capacities of the three plants, the demand requirements at the three sites, and the transportation costs per ton are as follows:

	Construction site			
Plant	**A**	**B**	**C**	**Supply (tons)**
1	**$ 8**	**$ 5**	**$ 6**	**120**
2	**15**	**10**	**12**	**80**
3	**3**	**9**	**10**	**80**
Demand (tons)	**150**	**70**	**100**	

Determine the linear programming model formulation and solve using Excel.

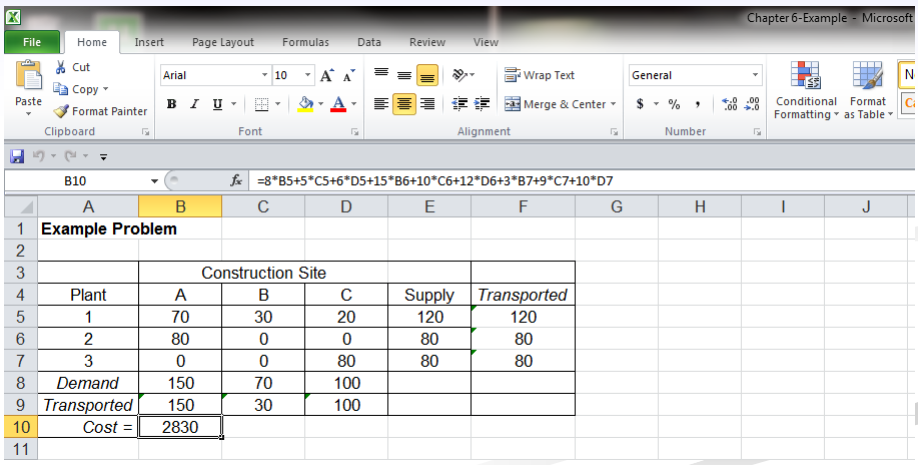
Example Problem Solution
Model Formulation

Minimize Z = $\$8x_{1A} + 5x_{1B} + 6x_{1C} + 15x_{2A} + 10x_{2B} + 12x_{2C} + 3x_{3A} + 9x_{3B} + 10x_{3C}$

subject to:

$$x_{1A} + x_{1B} + x_{1C} = 120$$
$$x_{2A} + x_{2B} + x_{2C} = 80$$
$$x_{3A} + x_{3B} + x_{3C} = 80$$
$$x_{1A} + x_{2A} + x_{3A} \leq 150$$
$$x_{1B} + x_{2B} + x_{3B} \leq 70$$
$$x_{1C} + x_{2C} + x_{3C} \leq 100$$
$$x_{ij} \geq 0$$

Example Problem Solution
Computer Solution with Excel



	A	B	C	D	E	F	G	H	I	J
1	**Example Problem**									
2										
3		Construction Site								
4	Plant	A	B	C	Supply	*Transported*				
5	1	70	30	20	120	120				
6	2	80	0	0	80	80				
7	3	0	0	80	80	80				
8	*Demand*	150	70	100						
9	*Transported*	150	30	100						
10	*Cost* =	2830								
11